Seat No. : _______________

DK-118

December-2017

M.Sc., Sem.-I

401 : Mathematics
(Mathematical Statistics – New)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.
(2) Normal Probability distribution tables are provided.
(3) Use of non-programmable scientific calculator is allowed.

1. (a) State (only) the Bayes' theorem.

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. Their statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.4, whereas this probability is 0.2 for a non accident-prone person. If we assume that 30 percent of the population is accident prone, **7**

(i) What is the probability that a new policyholder will have an accident within a year of purchasing a policy ?

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone ?

OR

State and prove the Chebyshev's inequality. State its usefulness.

Given mean and standard deviation as 8 and 1.5 respectively and the shape of the distribution unknown, determine an interval such that the probability is at least 8/9 that an observation will fall within that interval.

(b) A petrol pump is supplied with petrol once a day. If its daily volume X of sales in thousands of litres is distributed by $f(x) = 5\,(1 - x)^4, \quad 0 \le x \le 1$.

What must be the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01 (Use $\sqrt[5]{0.01} = 0.3981$) **7**

OR

Consider the following probability density function

$$f_x(x) = \begin{cases} 4x & 0 \le x \le \tfrac{1}{2} \\ 4(1-x) & \tfrac{1}{2} \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that it is indeed a pdf. Obtain the distribution function $f_x(x)$

2. (a) Find the distribution function associated with the pdf **7**

$$f_x(x) = \frac{x}{t^2} e^{-x^2/2t^2}, \ t > 0, x \geq 0$$
$$= 0, \text{ otherwise}$$

Consider the probability density function, $g_x(x) = k.\ \sin x$, $0 \leq x \leq \pi/2$. Find the appropriate value of k. Find the mean of the distribution.

OR

For what value of p is Var(X) maximum if the variate X has the distribution $P(X = 0) = P(X = 2) = p$; $P(X = 1) = 1 - 2p$, for $0 \leq p \leq 1/2$.

Show that the function defined by $f(x) = e^{-t}(1 - e^{-t})^{x-1}$, $t > 0$ can represent a probability function of a random variable X assuming the values 1, 2, 3,.... Find E(X).

(b) If X is a continuous random variable with probability density function f_x that satisfies $f_X(x) > 0$ for $a < x < b$, and if $y = H(x)$ is a continuous strictly increasing or strictly decreasing function of x, then show that the random variable $Y = H(X)$ has density function

$$f_Y(y) = f_X(x). \left|\frac{dx}{dy}\right|$$

with $x = H^{-1}(y)$ expressed in terms of y.

Suppose that the random variable X has the following density function

$$f_X(x) = \frac{x}{8} \qquad 0 \leq x \leq 4$$
$$= 0 \qquad \text{otherwise}$$

For the random variable $Y = H(X)$ where $H(x) = 2x + 8$, find the probability density function f_Y of Y **7**

OR

The probability density function of the random variables $[X_1, X_2]$ is given by

$$f(x_1, x_2) = \frac{k}{1000}, \qquad 0 \leq x_1 \leq 100, 0 \leq x_2 \leq 10$$
$$= 0, \qquad \text{otherwise}$$

Find the appropriate value of k, marginal densities of X_1 and X_2 and the expression for the cumulative distribution function $F(x_1, x_2)$

3. (a) State the pdf of a Poisson distribution. State its application. In standard notation, develop the Poisson distribution as a limiting form of the Binomial distribution. **7**

OR

For a random variable that follows the binomial distribution, find the first and the second moments about the origin and the second central moment. For a random variable that follows the gamma distribution, find the first and the second moments about the origin.

(b) In standard notations, derive the Mean and Variance of the Normal distribution. **7**

OR

Write the pdf of the Weibull distribution . State its various parameters. State its one important area of application.

The diameter of steel shafts is Weibull distributed with parameters $\gamma = 1.0$ inches, $\beta = 2$, and $\delta = 0.5$. Find the probability that a randomly selected shaft will not exceed 1.5 inches in diameter.

4. (a) State the pdf of the Beta distribution. Show that when both the shape parameters of the Beta distribution are 1, the Beta distribution reduces to the Uniform distribution. Graph the density function. Also show that when the parameters are (2, 1) or (1, 2), the Beta distribution reduces to the triangular probability distribution. Graph the density function. **7**

OR

State the pdf of Gamma distribution. State its mean and variance.

A redundant system operates with three units. Initially unit 1 is on line, while unit 2 and unit 3 are on standby. When unit 1 fails, the unit 2 is switched on until it fails and then unit 3 is switched on. The system life is represented as the sum of the subsystem lives. If the subsystem lives are independent of one another, and if the subsystems each have a life X_j, j = 1, 2, 3 having density $g(x) = \frac{1}{100} e^{-x/100}$, $x \geq 0$, then derive the reliability function of the total system.

(b) Explain the reproductive property of the Normal Distribution. **7**

An assembly consists of three linkage components X_1, X_2 and X_3 in series. Let $Y = X_1 + X_2 + X_3$. The properties of X_1, X_2 and X_3 are given below, with means in centimetres and variance in square centimetres.

$$X_1 \sim N(12, 0.02)$$

$$X_2 \sim N(24, 0.03)$$

$$X_3 \sim N(18, 0.04)$$

If X_1, X_2 and X_3 are independent, determine $P(53.8 \leq Y \leq 54.2)$

OR

X_1, X_2, X_3 and X_4 are independent random variables. Let $Y_1 = \ln X_1 \sim N(4, l)$, $Y_2 = \ln X_2 \sim N(3, 0.5)$, $Y_3 = \ln X_3 \sim N(2, 0.4)$, $Y_4 = \ln X_4 \sim N(1, 0.01)$.

For $W = e^{1.5}[X_1^{2.5} X_2^{0.2} X_3^{0.7} X_4^{3.1}]$, find $P(20{,}000 \leq W \leq 6{,}00{,}000)$

5. Attempt any **seven** questions : **14**

(1) Give the condition under which the DeMoivre-Laplace Approximation is fairly good.

(2) The random variable $Y=\ln X$ has a N(10,4) distribution. Find E(X) and the mode of the distribution of X.

(3) For the random variable X having a triangular probability density function as follows, compute $P_X(X \leq 3/2)$

$$f_X(x) = \begin{cases} x & 0 \leq x < 1 \\ 2 - x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(4) Write the probability mass function of the Pascal distributed random variable X. Write its mean and variance.

(5) An electric component is known to have a useful life represented by an exponential density with mean failure rate of 10^{-5} failures per hour. Find the percentage of such components that would fail before the mean life.

(6) Which are the distributions having the memoryless property ?

(7) The joint density of $[X_1, X_2]$ is given by

$$f(x_1, x_2) = \begin{cases} 6x_1 & 0 < x_1 < x_2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal of X_1 and the marginal of X_2

(8) For a random variable X, given the pdf $f(x) = \frac{1}{x\sqrt{2\pi}} e^{-\frac{(\log x)^2}{2}}$, $x > 0$, find E(X).

(9) State the Central Limit Theorem along with the necessary general conditions.

Cumulative Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.0	0.500 00	0.503 99	0.507 98	0.511 97	0.515 95	0.519 94	0.523 92	0.527 90	0.531 88	0.535 86	0.0
0.1	0.539 83	0.543 79	0.547 76	0.551.72	0.555 67	0.559 62	0.563 56	0.567 49	0.571 42	0.575 34	0.1
0.2	0.579 26	0.583 17	0.587 06	0.590 95	0.594 83	0.598 71	0.602 57	0.606 42	0.610 26	0.614 09	0.2
0.3	0.617 91	0.621 72	0.625 51	0.629 30	0.633 07	0.636 83	0.640 58	0.644 31	0.648 03	0.651 73	0.3
0.4	0.655 42	0.659 10	0.662 76	0.666 40	0.670 03	0.673 64	0.677 24	0.680 82	0.684 38	0.687 93	0.4
0.5	0.691 46	0.694 97	0.698 47	0.701 94	0.705 40	0.708 84	0.712 26	0.715 66	0.719 04	0.722 40	0.5
0.6	0.725 75	0.729 07	0.732 37	0.735 65	0.738 91	0.742 15	0.745 37	0.748 57	0.751 75	0.754 90	0.6
0.7	0.758 03	0.761 15	0.764 24	0.767 30	0.770 35	0.773 37	0.776 37	0.779 35	0.782 30	0.785 23	0.7
0.8	0.788 14	0.791 03	0.793 89	0.796 73	0.799 54	0.802 34	0.805 10	0.807 85	0.810 57	0.813 27	0.8
0.9	0.815 94	0.818 59	0.821 21	0.823 81	0.826 39	0.828 94	0.831 47	0.833 97	0.836 46	0.838 91	0.9
1.0	0.841 34	0.843 75	0.846 13	0.848 49	0.850 83	0.853 14	0.855 43	0.857 69	0.859 93	0.862 14	1.0
1.1	0.864 33	0.866 50	0.868 64	0.870 76	0.872 85	0.874 93	0.876 97	0.879 00	0.881 00	0.882 97	1.1
1.2	0.884 93	0.886 86	0.888 77	0.890 65	0.892 51	0.894 35	0.896 16	0.897 96	0.899 73	0.901 47	1.2
1.3	0.903 20	0.904 90	0.906 58	0.908 24	0.909 88	0.911 49	0.913 08	0.914 65	0.916 21	0.917 73	1.3
1.4	0.919 24	0.920 73	0.922 19	0.923 64	0.925 06	0.926 47	0.927 85	0.929 22	0.930 56	0.931 89	1.4
1.5	0.933 19	0.934 48	0.935 74	0.936 99	0.938 22	0.939 43	0.940 62	0.941 79	0.942 95	0.944 08	1.5
1.6	0.945 20	0.946 30	0.947 38	0.948 45	0.949 50	0.950 53	0.951 54	0.952 54	0.953 52	0.954 48	1.6
1.7	0.955 43	0.956 37	0.957 28	0.958 18	0.959 07	0.959 94	0.960 80	0.961 64	0.962 46	0.963 27	1.7
1.8	0.964 07	0.964 85	0.965 62	0.966 37	0.967 11	0.967 84	0.968 56	0.969 26	0.969 95	0.970 62	1.8
1.9	0.971 28	0.971 93	0.972 57	0.973 20	0.973 81	0.974 41	0.975 00	0.975 58	0.976 15	0.976 70	1.9
2.0	0.977 25	0.977 78	0.978 31	0.978 82	0.979 32	0.979 82	0.980 30	0.980 77	0.981 24	0.981 69	2.0
2.1	0.982 14	0.982 57	0.983 00	0.983 41	0.983 82	0.984 22	0.984 61	0.985 00	0.985 37	0.985 74	2.1
2.2	0.986 10	0.986 45	0.986 79	0.987 13	0.987 45	0.987 78	0.988 09	0.988 40	0.988 70	0.988 99	2.2
2.3	0.989 28	0.989 56	0.989 83	0.990 10	0.990 36	0.990 61	0.990 86	0.991 11	0.991 34	0.991 58	2.3
2.4	0.991 80	0.992 02	0.992 24	0.992 45	0.992 66	0.992 86	0.993 05	0.993 24	0.993 43	0.993 61	2.4
2.5	0.993 79	0.993 96	0.994 13	0.994 30	0.994 46	0.994 61	0.994 77	0.994 92	0.995 06	0.995 20	2.5
2.6	0.995 34	0.995 47	0.995 60	0.995 73	0.995 85	0.995 98	0.996 09	0.996 21	0.996 32	0.996 43	2.6
2.7	0.996 53	0.996 64	0.996 74	0.996 83	0.996 93	0.997 02	0.997 11	0.997 20	0.997 28	0.997 36	2.7
2.8	0.997 44	0.997 52	0.997 60	0.997 67	0.997 74	0.997 81	0.997 88	0.997 95	0.998 01	0.998 07	2.8
2.9	0.998 13	0.998 19	0.998 25	0.998 31	0.998 36	0.998 41	0.998 46	0.998 51	0.998 56	0.998 61	2.9
3.0	0.998 65	0.998 69	0.998 74	0.998 78	0.998 82	0.998 86	0.998 89	0.998 93	0.998 97	0.999 00	3.0
3.1	0.999 03	0.999 06	0.999 10	0.999 13	0.999 16	0.999 18	0.999 21	0.999 24	0.999 26	0.999 29	3.1
3.2	0.999 31	0.999 34	0.999 36	0.999 38	0.999 40	0.999 42	0.999 44	0.999 46	0.999 48	0.999 50	3.2
3.3	0.999 52	0.999 53	0.999 55	0.999 57	0.999 58	0.999 60	0.999 61	0.999 62	0.999 64	0.999 65	3.3
3.4	0.999 66	0.999 68	0.999 69	0.999 70	0.999 71	0.999 72	0.999 73	0.999 74	0.999 75	0.999 76	3.4
3.5	0.999 77	0.999 78	0.999 78	0.999 79	0.999 80	0.999 81	0.999 81	0.999 82	0.999 83	0.999 83	3.5
3.6	0.999 84	0.999 85	0.999 85	0.999 86	0.999 86	0.999 87	0.999 87	0.999 88	0.999 88	0.999 89	3.6
3.7	0.999 89	0.999 90	0.999 90	0.999 90	0.999 91	0.999 91	0.999 92	0.999 92	0.999 92	0.999 92	3.7
3.8	0.999 93	0.999 93	0.999 93	0.999 94	0.999 94	0.999 94	0.999 94	0.999 95	0.999 95	0.999 95	3.8
3.9	0.999 95	0.999 95	0.999 96	0.999 96	0.999 96	0.999 96	0.999 96	0.999 96	0.999 97	0.999 97	3.9

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DK-118

December-2017

M.Sc., Sem.-I

401 : Mathematics
(Functions of Several Variables – 1 (Old))

Time : 3 Hours] **[Max. Marks : 70**

1. (a) Attempt any **one** : **7**
   (i) Define the convex set. Show that the intersection of two convex sets is convex. Is this true for the union ? Justify.
   (ii) State and prove the Cauchy's inequality.

   (b) Attempt any **two** : **4**
   (i) Define r-simplex. Give the sketch of a 0-simplex, 1-simplex and a 2-simplex.
   (ii) Show that $K = \{(x, y)/x^2 + y^2 \le 1\}$ is a convex subset of E^2. Is it closed ? Justify,
   (iii) Define a convex function. Is the function $f(x) = x - x^2$ convex on $\mathbb{R}$? Justify.

   (c) Answer very briefly : **3**
   (i) Give two non-trivial real linear maps defined on E^3.
   (ii) Define hyperplane in E^n. Define the closed half space.
   (iii) If x_1 and x_2 are two non-zero vectors in E^n such that the inner product $x_1 . x_2 = 0$, prove that $\{x_1, x_2\}$ is a linearly independent set.

2. (a) Attempt any **one** : **7**
   (i) Define the directional derivative of f at x_0 in the direction v. Let $f(x, y) = (xy)^{\frac{1}{3}}$. Find all the directions in which the derivative of f exist at (0,0).
   (ii) Define the tangent plane to f at $(x_0, f(x_0))$. Let $f(x, y) = 3x^2y + 2xy^2$. Find the tangent plane at (l, –2, 2).

   (b) Attempt any **two** : **4**
   (i) Let $f(x, y) = \dfrac{2xy^2}{x^2 + y^4}$ if $(x, y) \ne 0$ and $f(0, 0) = 0$. Show that f is not differentiable at (0, 0).
   (ii) Define the $C^{(k)}$ function. Give an example of a $C^{(1)}$ function that is not $C^{(2)}$.
   (iii) Give an example of a differentiable function $f : \mathbb{R} \to \mathbb{R}$ that is not $C^{(1)}$.

(c) Answer very briefly : **3**

(i) If f is of class $C^{(1)}$ then $f_{ij} = f_{ji}$ for all $i \neq j$. True or false ? Justify your answer.

(ii) Show that the derivative of f at x_0 in the direction $-\upsilon$ is the negative of the derivative of f at x_0 in the direction υ.

(iii) Give an example of a nowhere continuous function on R.

3. (a) Attempt any **one** : **7**

(i) Evaluate $\int x dy - y dx$ over the curve γ, where γ is the triangle in E^2 with vertices (0, 0), (b, c) and (a, 0).

(ii) Evaluate $\int x dy - y dx$ over the curve γ, where γ is the semi-circle with centre (0, 0) and endpoints ae_2 and $-ae_2$, directed from $-ae_2$ to ae_2.

(b) Attempt any **two** : **4**

(i) Let $g(t) = (\cos 2t)e_1 + (\sin 2t)e_2$, $0 \leq t \leq 4\pi$. Find the trace of the curve represented by g. Is the curve simple, closed ?

(ii) For any closed curve γ lying in D and for any continuous exact 1-forrn ω, show that $\int_\gamma \omega = 0$.

(iii) Let $D \subset E^2$ be open and simply connected, and u, v are functions of class $C^{(1)}$ which satisfy Cauchy-Riemann equations. Show that for any closed curve γ lying in D, $\int_\gamma v dx + u dy = 0$.

(c) Answer very briefly : **3**

(i) Define the parametric representation of class $C^{(1)}$.

(ii) Define the multiplicity of a point and the trace of a curve γ.

(iii) Let $g(t) = (\cos \theta)e_1 + (\sin \theta)e_2 + \theta e_3$, where $0 \leq \theta \leq 2\pi$, identify the curve.

4. (a) Attempt any **one** : **7**

(i) Let $g(s, t) = |s - t| e_1 + |s + t| e_2$, $\Delta = E^2$. Find $g(E^2)$. Find the inverse image of any line $y = mx$ through the origin.

(ii) Let n = r = 3, and let L be the linear transformation that takes the standard basis vectors $\in_1, \in_2, \in_3$ respectively to $v_1 = e_1 + 2e_2 - e_3$, $v_2 = -e_1 + e_2$, $v_3 = -e_1 + 4e_2 - e_3$. Find the matrix of L, the rank, and the kernel.

(b) Attempt any **two** : **4**

(i) Let $F(x, y) = f(x, xy)$. Express the mixed partial derivative F_{12}.

(ii) Find the Jacobian of $g(s, t) = (s^2 - t^2)e_1 + (2st)e_2$, where $\Delta = E^2$.

(iii) State (only) the Inverse function theorem.

(c) Answer very briefly : **3**

(i) Define (a) isometry and (b) rotation.

(ii) Give an example of a vector space of dimension 8 over E^1.

(iii) Let L be the linear tranfromation from E^2 to E^2 that rotates the plane about the origin by $\frac{\pi}{4}$ angle. Find the matrix of L.

5. (a) Attempt any **one** : **7**

(i) State (without proof) the Implicit function theorem. Explain the theorem with a simple illustraion.

(ii) Does $g(s, t) = (s^2 - 2 - 2)e_1 + 3te_2$, $\Delta = E^2$ satisfy the hyotheses of the inverse function theorem ? Find $g(\Delta)$. Find g^{-1}, if g is univalent.

(b) Attempt any **two** : **4**

(i) Let $g(t) = t + x_0$, $\Delta = E^n$, Find $g(\Delta)$ and g^{-1}.

(ii) What is an r-manifold ? Explain r-manifold by taking r = l or 2.

(iii) Define (a) regular transformation (b) function of class C^1.

(c) Answer very briefly : **3**

(i) When we say that $f : E^n \rightarrow E^1$ is differentiable at point x_0 ?

(ii) Show that (0, 1) is homeomorphic to E^1.

(iii) Show that the closed interval [0, 1] is not a 1-manifold.